University of Turku / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 05, 5.10.2018

`Problem sessions will be held on Fridays at 8-10`
N.B. The files mentioned in the exercises (if any) are available on the course homepage. **The exam problems** will be mostly writing on pen and paper short MATLAB scripts. You may bring to exam one A4 ("lunttilappu") containing useful information.

1. (a) The program hlp051.m produces a picture of a well-known distribution. What is this distribution?

(b) The function $erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-y^2)dy$ is a built-in function of MATLAB. Show by MATLAB experiments that for $a, b \in R, a \neq 0, x_1 < x_2$,

$$\int_{x_1}^{x_2} \exp\left(-\frac{(x-b)^2}{(2a^2)}\right) dx = a\sqrt{\frac{\pi}{2}}\left(erf(\frac{x_2-b}{a\sqrt{2}}) - erf(\frac{x_1-b}{a\sqrt{2}})\right) .$$

This can also be verified by change of variable.

2. On the www-page is given the program `hpl052.m` which compares two methods of numerical integration, namely Riemann's sum and Simpson's Rule, over a rectangular region in the plane with the test function $f(x, y) = xy$. The program prints the error.

(a) Modify the program to use the function $g(x, y) = \sin(2x) * \cos(4 * y)$ and report the results.

(b) Write the code also for the Trapezoid Rule and one of MATLAB's built-in function `dblquad, quadl, integral,...` and report the error. Provide an order or preference of the methods based on the accuracy of each method.

3. Generate N pairs of points A, B in the unit disk $\{(x, y) : x^2 + y^2 < 1\}$ and print the mean value of the distances. Repeat this for $N = 1000 : 1000 : 30000$.

4. Suppose that A is a non-singular $n \times n$ matrix with columns $A^{(j)}, j = 1, ..., n$, and x and b are $n \times 1$ vectors. By Cramer's Rule, the solution to $Ax = b$ is given by

$$x_j = ((\det(A))^{-1}) \det(C_j) , C_j = [A^{(1)} A^{(2)} ... A^{(j-1)} b A^{(j+1)} ... A^{(n)}] .$$

Verify this procedure with MATLAB tests for small n. For how big values of n this is a reasonable procedure?

5. The capacity of an oil container is 3000 l and its shape is a right circular cylinder. The cross section is a circle with diameter 1.3 m. Find the height of the container. The symmetry axis of the container is horizontal. Write a table of the form

```
    n       Measurement/m    Amount of oil/1000 l
-----------------------------------------------
    1       0                0
    2       0.05             0.0379
 ......

   14       0.65             1.5
```

Here the measurement column indicates how much oil is left in the container in meters and the next column indicates the volume of the oil. What is the measurement when 1000 l oil is remaining?

6. Polynomial Inequalities Springer-Verlag, 1995 states that if $p(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_0$ and $a_0 \geq a_1 \geq ... \geq a_n > 0$ then all zeros of p lies outside the open unit disk. Verify experimentally this statement by generating random coefficients a_j and by plotting the roots in the plane. Polynomials of this type occur e.g. in the analysis of time series.