University of Turku / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 6, 12.10.2018

ACHTUNG. Jos olet palauttanut demoja emailitse, on tärkeää, että lähetät koosteen palautuspäivistä ja ratkaisujen lukumääristä 13.10.2018 mennessä os. `vuorinen@utu.fi`. Muuten ei ole takeita siitä, että kirjapidot täsmäävät. Huomaa erityisesti tämän harjoituksen tehtävä 4.

Tentit: to 25.10.2018, ma 12.11 ja ma 3.12 klo 9-12, IX, X ks. OpintoOpas

Palaute: Täytäthän opetuksen arviointikaavakkeen. Palauta se nimettömänä luennoitsijalle tai kansliaan. Lomakkeita jaossa demoissa, luennolla, kurssin www-sivulla ja tehtävälokerikossa.

N.B. The files mentioned in the exercises (if any) are available on the course homepage

1. The h-index is an index that attempts to measure both the productivity and impact of the published work of a scientist or scholar. The index is based on the set of the scientist's most cited papers and the number of citations that they have received in other publications (see http://en.wikipedia.org/wiki/H-index) The index is based on the distribution of citations received by a given researcher's publications. Hirsch writes:

"A scientist has index h if h of his/her N papers have at least h citations each, and the other $(N-h)$ papers have no more than h citations each."

(a) Suppose that Dr. Smart has 50 papers and paper $j, j = 1, ..., 50$ is cited c_j times. Suppose that $c_j = 1 + fix(60 * rand)$ for all $j, j = 1, ..., 50$. Find the h-index.

(b) Dr. Bright has written excellent 50 papers and the correponding citation counts are $c_j = j^p, j = 1, ..., 50$ for some $p > 0$. Find the corresponding index h_p and also plot h_p as a function of p when $p = 0.2 * m, m = 1, ..., 20$.

2. The daily temperature data is observed and the results appear in the table below. Create a file with these thirteen (x,y) pairs of this temperature data.

x	0.0	2.0	4.0	6.0	8.0	10.0	12.0	14.0	16.0	18.0	20.0	22.0	24.0
y	6.3	4.0	6.6	10.9	14.6	19.1	24.3	25.7	22.9	19.5	15.9	10.3	5.4

FILE: ~/MME2018/d06/d06.tex — 9. lokakuuta 2018 (klo 16.07).

Let $m = \min\{y\}, M = \max\{y\}$ and set $a = 0.5*(M+m), b = 0.5*(M-m)$. Try to find some reasonable integer value for the parameter c in the interval $[0, 24]$ so that the curve $y = a + b*\sin(2*\pi*(x-c)/24))$ becomes as close to the data as possible. Carry out the following steps:

(a) Read the data $(x_j, y_j), j = 1, ..., 13$, from the file [or copy these values in a vector] and compute the maximum and minimum temperatures M and m. Then compute a and b.

(b) For each $c = 0 : 24$ compute

$$A(c) = \sum_{j=1}^{13}(y_j - (a + b*\sin(2*\pi*(x_j - c)/24)))^2 ,$$

and choose the value of c that yields the minimal $A(c)$.

(c) With these values of the parameters a, b, c plot the curve $y = a + b*\sin(2*\pi*(x-c)/24))$ and the data points in the same picture.

3. To fit a circle (1) $(x - c_1)^2 + (y - c_2)^2 = r^2$ to n sample pairs of coordinates $(x_k, y_k), k = 1, ..., n$ we must determine the center (c_1, c_2) and the radius r. Now (1) $\Leftrightarrow$ (2) $2xc_1 + 2yc_2 + (r^2 - c_1^2 - c_2^2) = x^2 + y^2$. If we set $c_3 = r^2 - c_1^2 - c_2^2$, then the equation takes the form

$$2xc_1 + 2yc_2 + c_3 = x^2 + y^2 .$$

Substituting each data point we get

$$\begin{bmatrix} 2x_1 & 2y_1 & 1 \\ & \vdots & \\ 2x_n & 2y_n & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} x_1^2 + y_1^2 \\ \vdots \\ x_n^2 + y_n^2 \end{bmatrix}$$

This system can be solved in the LSQ-sense (if a is $m \times n, m > n$, then the LSQ solution of $a * x = b$ is given by

```
{ x = a \ b }
```

Then $r = \sqrt{c_3 + c_1^2 + c_2^2}$. Apply this algorithm for the points generated by

```
s=0.5+0.5*rand(10,1);
theta=2*pi*rand(10,1);
clear x
clear y
x=3*s.*cos(theta) ;
y=3*s.*sin(theta);
```

Plot the data and the fitted circle.

4. (a) The number of returned home works of the weekly problem sessions of the 1998 scientific computing course during the first six weeks were 21, 24, 17, 21, 14 and 17, respectively. Fit a model of the form

$$y = \lambda_1 \exp(-\lambda_2 x)$$

to this data and predict the number of participants in the 12th problem session.

(b) In 2018 in a similar course, during the first 5problem sessions these activity numbers were 23, 20, 19, 20, 16. Use this data to predict the number active students for week 6.

5. Recall from linear algebra that if $\Delta \equiv ad - bc \neq 0$, then

$$\Delta \begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

Consider the Newton method $x_{n+1} = x_n - J_f(x_n)^{-1} f(x_n)$ to solve the system

$$\begin{cases} x_1^2 + x_2^2 - 1 & = 0 \\ x_1^2 - x_2^2 & = 0 \end{cases}$$

with the initial point $(0.5, 0.5)$. Use standard methods from linear algebra and multidimensional calculus, such as the above formula for matrix inverse, to compute the following expressions analytically and to print the numerical values $J_f(x_n)$, $J_f(x_n)^{-1}, T(x_n) \equiv J_f(x_n)^{-1} f(x_n)$ for the first three iteration steps.

6. The program hlp066new.m on the www-page plots a curve through given points. Use it to plot the shape of your hand using sufficiently many points, e.g.25-30 points. Experiment with the program by changing pchip to spline and other methods of interpolation.