University of Turku / Department of Mathematics and Statistics
SCIENTIFIC COMPUTING
Exercise 01 / Solutions

1. Apply the recursion formula $x_0 = 1, x_{n+1} = \frac{1}{2}(x_n + \frac{a}{x_n}), n = 0, 1, 2, \ldots$ for $\sqrt{a}$ to compute $\sqrt{3}$. Print the results in the following format:

```
n  x(n)   Error
0  1
.....
6  ...
```

Solution:

```
% FILE d011 .m begins.
x=1; a=3;
fprintf('    n     x(n)            Error\n')
for j=1:6
%    disp([j-1, x, x-sqrt(3)]);
    fprintf('   %2d%16.10f   %12.4e\n',j-1,x,x-sqrt(3));
    x=0.5*(x+a/x);
end
% FILE d011.m ends.
```

Output:

```
    n      x(n)            Error
    0     1.0000000000    -7.3205e-01
    1     2.0000000000     2.6795e-01
    2     1.7500000000     1.7949e-02
    3     1.7321428571     9.2050e-05
    4     1.7320508100     2.4459e-09
    5     1.7320508076     0.0000e+00
```

2. Approximations to the number π are given by the formula

$$p(n) = \sum_{k=0}^{n} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right).$$

Print the first few results in the same format as in problem 1.

Solution:

```
% FILE d012.m begins.
s=0;
fprintf('%3s  %12s  %12s\n', 'n', 'p(n)', 'Error')
for k=0:10
    fprintf('%3d  %12.8f  %12.4e\n',k,s,s-pi);
    s=s+(16^(-k))*(4/(8*k+1)- 2/(8*k+4)-1/(8*k+5)-1/(8*k+6));
end
% FILE d012.m ends.
```

Output:

```
  n          p(n)         Error
  0    0.00000000   -3.1416e+00
  1    3.13333333   -8.2593e-03
  2    3.14142247   -1.7019e-04
  3    3.14158739   -5.2632e-06
  4    3.14159246   -1.9602e-07
  5    3.14159265   -8.1295e-09
  6    3.14159265   -3.6171e-10
  7    3.14159265   -1.6912e-11
  8    3.14159265   -8.2023e-13
  9    3.14159265   -4.0856e-14
 10    3.14159265   -1.7764e-15
```

3. According to an Internet page, the center w of a circle through three points a, b, c in the complex plane can be found as follows in MATLAB notation where conj is the complex conjugate:

```
u=(b-c).*abs(a).^2 + (c-a).*abs(b).^2 + (a-b).*abs(c).^2;
v=(b-c).*conj(a)+  (c-a).*conj(b)+  (a-b).*conj(c);
w= u./v;
```

Write a MATLAB script to check this claim. (Hint: Take three random points on the unit circle, then compute w and show that it is 0.)

Solution:

```
%FILE:  d013.m begin
%
function w=noname
clear
close all
```

```
a=exp(i*pi*2*rand);
b=exp(i*pi*2*rand);
c=exp(i*pi*2*rand);
w=cent(a,b,c);
disp(w)
close all
figure

plot(real(exp(i*(0:0.01*pi:2*pi))), imag(exp(i*(0:0.01*pi:2*pi))),'k-')
hold on
plot([a,b,c,w],'k*','MarkerSize',10)
axis equal
%print -dps fig013.ps

function w=cent(a,b,c)
% a,b,c are complex numbers.  cent returns w,
% the center of the circle through all of them
u=(b-c).*abs(a).^2 + (c-a).*abs(b).^2 + (a-b).*abs(c).^2 ;
v=(b-c).*conj(a)+ (c-a).*conj(b) + (a-b).*conj(c) ;
w=u./v;

%FILE:  d013.m end
```

Output:

```
     0


ans =

     0
```

4. Let $(x_j, y_j), j = 0, 1, ..., n$ be the vertices of a polygon with $(x_0, y_0) = (x_n, y_n)$. The area of the polygon is given by $a = \frac{1}{2}\sum_{i=1}^{n} t_i$ with $t_i = x_{i-1}y_i - x_i y_{i-1}$. Carry out the following steps for each of the regular polygons triangle, square and hexagon:

(a) Choose vertices and compute the area by school geometry.

(b) Compute the area by the formula and compare to the exact value.

(c) Plot the figure.

Solution:

```
% FILE widemarg.m begins.
% Creates wide margins for figure n
function widemarg(n)
figure(n)
ax=axis;
dx=0.07*(ax(2)- ax(1));
dy=0.07*(ax(4)- ax(3));
newax=[ ax(1)-dx ax(2)+dx ax(3)-dy ax(4)+dy];
axis(newax);

% FILE widemarg.m ends.

% FILE d014.m begins.
% USES: widemarg.m
path(path,'../util')
close all
disp('Triangle')
x=[0, 2, 1, 0]; y=[0, 0, sqrt(3), 0];
fprintf('d014area = %12.5e  , exact area = %12.5e \n', ...
```

```
d014area(x,y),sqrt(3))
%figure
subplot(2,2,1)
plot(x,y)
widemarg(gcf)
patch(x,y,'y')
disp('Square')
x=[0, 2, 2, 0, 0]; y=[0, 0, 2, 2, 0];
fprintf('d014area = %12.5e  , exact area = %12.5e \n', ...
d014area(x,y),4)
%figure
subplot(2,2,2)
plot(x,y)
widemarg(gcf)
patch(x,y,'y')
disp('Hexagon')
k=0:6;
x=real(exp(i*pi*k/3)); y=imag(exp(i*pi*k/3));
fprintf('d014area = %12.5e  , exact area = %12.5e \n', ...
d014area(x,y),1.5*sqrt(3))
%figure
subplot(2,2,3)
plot(x,y)
widemarg(gcf)
patch(x,y,'y')
print -dps d014.ps
% FILE d014.m ends.
```

Output:

```
Triangle
d014area =  1.73205e+00   , exact area =  1.73205e+00
Square
d014area =  4.00000e+00   , exact area =  4.00000e+00
Hexagon
d014area =  2.59808e+00   , exact area =  2.59808e+00
```

5. Hilbert's inequality says that for $a_k, b_k \geq 0$

$$\sum_{m=0}^{\infty}\sum_{n=0}^{\infty}\frac{a_m b_n}{m+n+1} \leq \pi(\sum_{m=0}^{\infty} a_m^2)^{1/2}(\sum_{n=0}^{\infty} b_n^2)^{1/2} .$$

Carry out a numerical verification of this inequality.

Solution:

```
% FILE d015.m begins.
% Use random numbers such that the RHS series converge
% Choose mmax terms for each sum, mmax large
% Use terms a_k = rand *1/k, b_k = rand *1/k
close all
for tst=1: 10     % counter for test number
mmax=1000+fix(10000*rand);
a= (1:mmax); a=(1./a).*rand(1,mmax);
b= (1:mmax); b=(1./b).*rand(1,mmax);
rhs1=sum(a.^2); rhs2=sum(b.^2);
lhs=0;           %The values of the left hand side
                 % will be accumulated in this variable
for ii=1:mmax
for jj=1:mmax
lhs=lhs+ a(ii)*b(jj)/((ii-1)+(jj-1) +1);   % korjattu 14.9.2009
end              % end of jj loop
end              % end of ii loop
rhs=pi*sqrt(rhs1*rhs2); % The right hand side of the inequality
fprintf('%2d. Test with %6d  terms: RHS-LHS = %g\n',tst,mmax,rhs-lhs)
end              % end of tst loop
% FILE d015.m ends.
```

Output:

```
 1. Test with   6042  terms: RHS-LHS = 0.529862
 2. Test with  10843  terms: RHS-LHS = 0.540741
 3. Test with   3166  terms: RHS-LHS = 0.761626
 4. Test with   5252  terms: RHS-LHS = 1.09569
 5. Test with   8986  terms: RHS-LHS = 0.760386
 6. Test with  10155  terms: RHS-LHS = 1.1506
 7. Test with   1132  terms: RHS-LHS = 1.04392
 8. Test with   6501  terms: RHS-LHS = 0.880126
 9. Test with   8917  terms: RHS-LHS = 1.01719
10. Test with   3031  terms: RHS-LHS = 0.899637
```

6. What does the following program do? Execute it and interprete the results.

```
% FILE d016.m begins.
for pp=1:3
a=2*rand; b=3*(a+1);
f=@(x)(a*sin(b*x)); v=quad(f,0,1);
exact=(a/b)*(1-cos(b));
fprintf(' %6.4f  %6.4f  %12.6f   %12.4e\n', a, b, v, v-exact)
end
% FILE d016.m ends.
```

Solution:

```
% FILE d016.m begins.
for pp=1:3
a=2*rand; b=3*(a+1);
f=@(x)(a*sin(b*x)); v=integral(f,0,1);
%older version MATLAB: v=quad(f,0,1);
exact=(a/b)*(1-cos(b));
fprintf(' %6.4f  %6.4f  %12.6f   %12.4e\n', a, b, v, v-exact)
end
% FILE d016.m ends.
```

Output:

```
 0.7568  5.2704      0.067559    -5.5511e-17
 0.4795  4.4385      0.137252    -2.7756e-17
 1.1315  6.3945      0.001095     1.7347e-17
```